Bader Written Calculation Policy

This policy details the progression in written calculation methods from Year 1 to Year 6. The methods in this policy will provide children with the skills required by the National Curriculum.

This policy details written methods only. At Bader Primary School, there is a renewed emphasis on developing children's confidence and proficiency in mental calculation and mental arithmetic. It is through understanding how to manipulate number that children develop their conceptual understanding.

Proficiency and accuracy in the methods contained in this policy will ensure that children develop procedural fluency. The CPA (concrete -> pictorial -> abstract) approach will support children's understanding of the methods they are taught. It is important that all children follow this approach so that fluency is based on understanding rather than memorising a procedure.

Children should be encouraged to use mental methods or jottings where appropriate

Readiness for formal written methods

- know the place value of digits in whole numbers and decimals
- know by heart all addition and subtraction facts for numbers up to 20
- partition a number in different ways
- recall appropriate multiplication facts
- use known facts and place value to multiply and divide mentally
- confident using written subtraction
- understand zero as a place holder
- know the approximate size of the answer

Progression in the teaching of counting in Foundation Stage

Pre-counting The key focus in pre-counting is an understanding of the concepts more, less and the same and an appreciation of how these are related. Children at this stage develop these concepts by comparison and no counting is involved.	**Ordering** Count by reciting the number names in order forwards and backwards from any starting point.	**One to one correspondence** One number word has to be matched to each and every object. Lack of coordination is a source of potential error – it helps if children move the objects as they count, use large rhythmic movements, or clap as they count.	**Cardinality (Knowing the final number counted is the total number of objects)** Count out a number of objects from a larger collection. Know the number they stop counting at will give the total number of objects.
Pre-counting ideas *Provide children with opportunities to sort groups of objects explicitly using the language of* ***more*** *and* ***less.***   *Which group of apples has the most?* *Which group of apples has the least?*	**Ordering ideas** *Provide children with opportunities to count orally on a daily basis. Rote count so that children are able to understand number order and can hear the rhythm and pattern. Use a drum or clap to keep the beat.* 	**One to one correspondence ideas** *Play counting games together moving along a track, play games involving amounts such as knocking down skittles.* *Use traditional counting songs throughout the day ensuring children have the visual/kinaesthetic resources eg. 5 little ducks, 10 green bottles* 	**Cardinal counting ideas**  *How many bananas are in my fruit bowl? Allow children to physically handle the fruit.* *Provide children with objects to point to and move as they count and say the numbers.*

Progression in the teaching of counting in Foundation Stage

Subitising (recognise small numbers without counting them)

Children need to recognise small amounts without counting them eg. dot patterns on dice, dots on tens frames, dominoes and playing cards as well as small groups of randomly arranged shapes stuck on cards.

Abstraction

You can count anything – visible objects, hidden objects, imaginary objects, sounds etc. Children find it harder to count things they cannot move (because the objects are fixed), touch (they are at a distance), see, that move around. Children also find it difficult to count a mix of different objects, or similar objects of very different sizes.

Conservation of number – MASTERY!

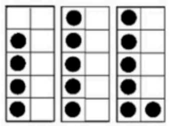
Ultimately children need to realise that when objects are rearranged the number of them stays the same.

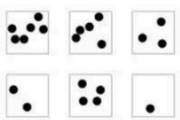
End of year counting expectations

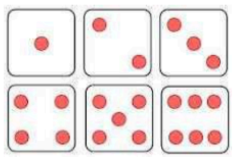
- count reliably to 20
- count reliably up to 10 everyday objects
- estimate a number of objects then check by counting
- use ordinal numbers in context eg first, second, third
- count in twos, fives and tens
- order numbers 1-20
- say 1 more/ 1 less than a given number to 20

Subitising ideas

Provide children with opportunities to count by recognising amounts.



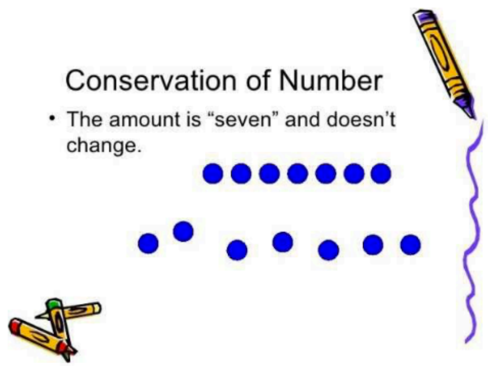




Abstraction ideas

How many pigs are in this picture?

Provide children with a variety of objects to count.



Addition

Whole Numbers

Year 1

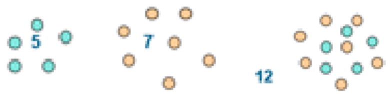
+ = signs and missing numbers
Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

2 = 1+ 1
2 + 3 = 4 + 1

Missing numbers need to be placed in all possible places.

3 + 4 = = 3 + 4
3 + = 7 7 = + 4

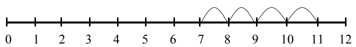
Counting and Combining sets of Objects
Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)



Understanding of counting on with a numbertrack.

Understanding of counting on with a numberline
(supported by models and images).

7+ 4



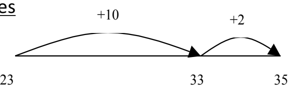
Year 2

Missing number problems e.g 14 + 5 = 10 + __ 32 +_ + __= 100
35 = 1 +__ + 5

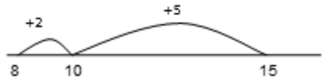
It is valuable to use a range of representations (also see Y1).
Continue to use numberlines to develop understanding of:

Counting on in tens and ones

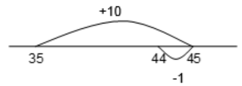
23 + 12 = 23 + 10 + 2
= 33 + 2
= 35



Partitioning and bridging through 10.
The steps in addition often bridge through a multiple of 10
e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.
8 + 7 = 15

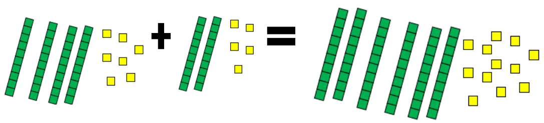


Adding 9 or 11 by adding 10 and adjusting by 1
e.g. Add 9 by adding 10 and adjusting by 1
35 + 9 = 44

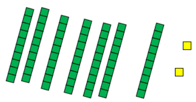


Towards a Written Method
Partitioning in different ways and recombine
47+25

47 **25** **60 + 12**



Leading to exchanging:
72



Expanded written method
40 + 7 + 20 + 5 =
40+20 + 7 + 5 =
60 + 12 = 72

40 + 7
+ 20 + 5
60 + 12 = 72

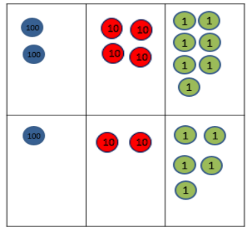
Year 3

Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones
Partition both numbers and recombine.
Count on by partitioning the second number only e.g.
247 + 125 = 247 + 100 + 20+ 5
= 347 + 20 + 5
= 367 + 5
= 372

Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

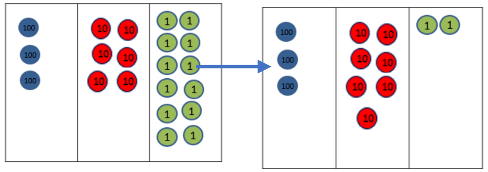
Towards a Written Method
Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)



200 + 40 + 7
100 + 20 + 5
300 + 60 +12 = 372

  247
+125
   12
   60
  300
  372

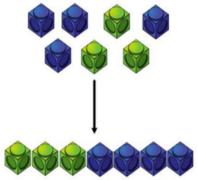
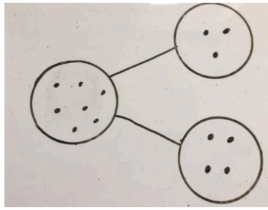
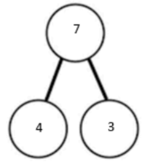
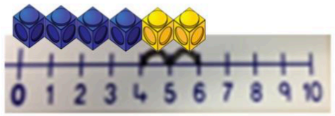
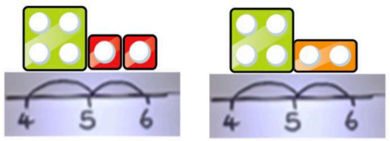
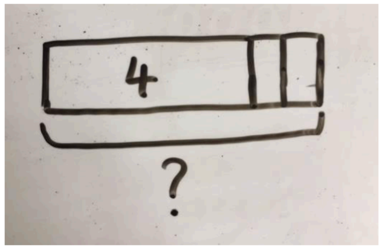
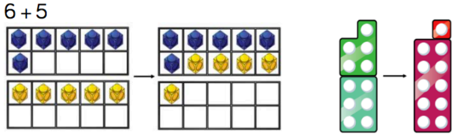
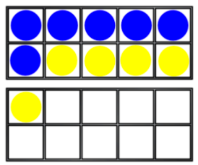
Leading to children understanding the exchange between tens and ones.



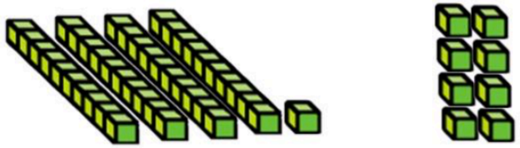
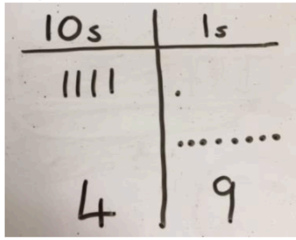
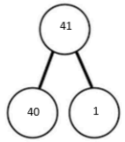
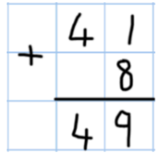
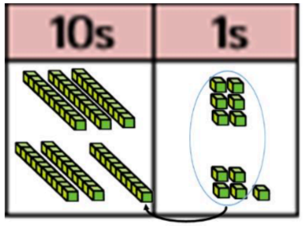
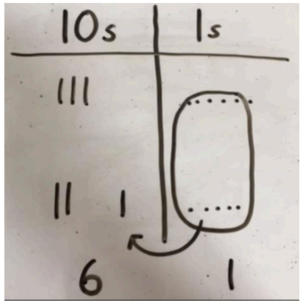
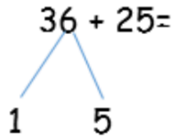
Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

  247
+125
  372
   10

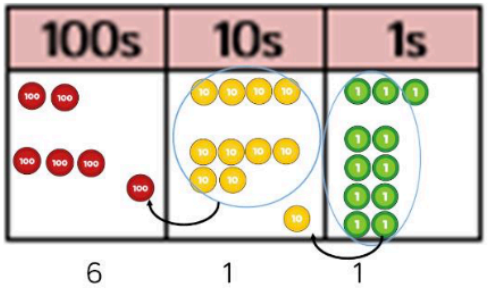
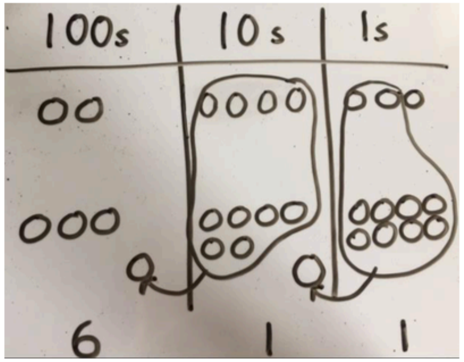
Year 1 Concrete – Pictorial - Abstract

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 
Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4

Year 2 Concrete – Pictorial - Abstract

TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25  6 1	Chidlren to represent the base 10 in a place value chart. 	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61 1

Year 3 and beyond Concrete – Pictorial - Abstract

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.  	Chidren to represent the counters in a place value chart, circling when they make an exchange.  	$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$

Addition
Whole Numbers

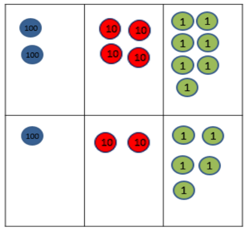
Year 4

Missing number/digit problems:

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to 4-digits)

Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.

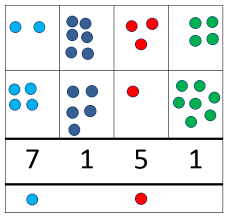


```
200 + 40 + 7
100 + 20 + 5
300 + 60 +12 = 372

  247
 +125
   12
   60
  300
  372
```

Compact written method

Extend to numbers with at least four digits.



```
 2634
+4517
 7151
  1 1
```

Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

```
  72.8
+ 54.6
 127.4
 1 1
```

Year 5

Missing number/digit problems:

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency e.g. 12462 + 2300 = 14762

Written methods (progressing to more than 4-digits)

As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

```
  172.83
+  54.68
  227.51
  1  1 1
```

Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

Year 6

Missing number/digit problems:

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue calculating with decimals, including those with different numbers of decimal places

Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.

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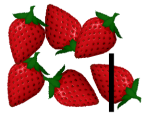
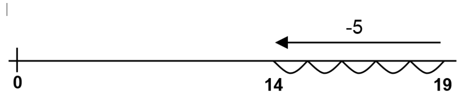
Subtraction
Whole Numbers

Year 1

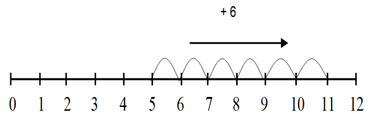
Missing number problems e.g. 7 = □ - 9; 20 - □ = 9; 15 – 9 = □; □ - □ = 11; 16 – 0 = □
Use concrete objects and pictorial representations. If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown.

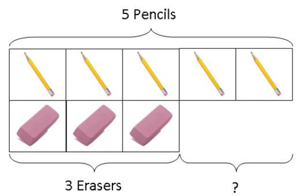
Understand subtraction as take-away:





Understand subtraction as finding the difference:

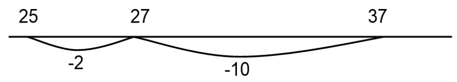


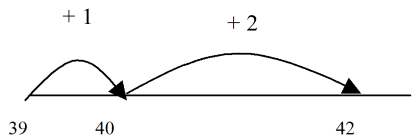


The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.
The use of other images is also valuable for modelling subtraction e.g. Ten frames, Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

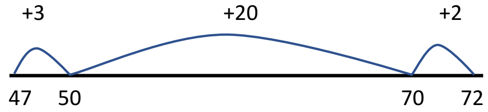
Year 2

Missing number problems e.g. 52 – 8 = □; □ – 20 = 25; 22 = □ – 21; 6 + □ + 3 = 11
It is valuable to use a range of representations (also see Y1). Continue to use number lines to model take-away and difference. E.g.

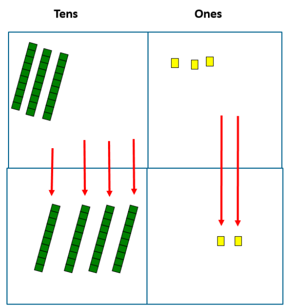




The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



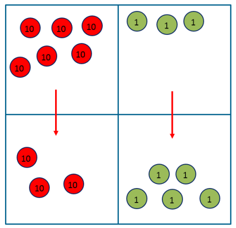
The bar model should continue to be used, as well as images in the context of **measures**.
Towards written methods
Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. 75 – 42



	70	5
-	40	2
	30	3

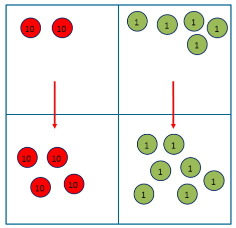
Year 3

Missing number problems e.g. □ = 43 – 27; 145 – □ = 138; 274 – 30 = □; 245 – □ = 195; 532 – 200 = □; 364 – 153 = □
Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving (see Y1 and Y2).
Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.
Written methods (progressing to 3-digits)
Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation)



	90	8
-	30	5
	60	3

For some children this will lead to exchanging, modelled using place value counters (or Dienes).

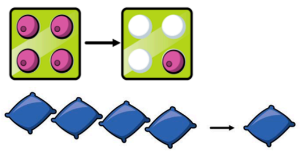
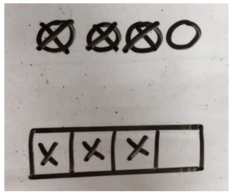
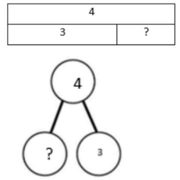
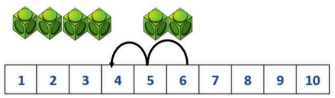
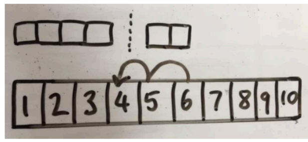
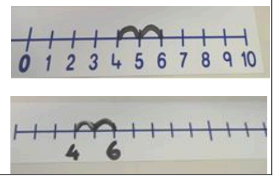
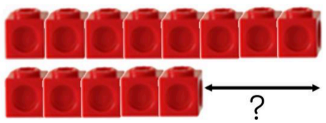
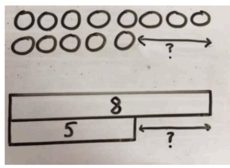
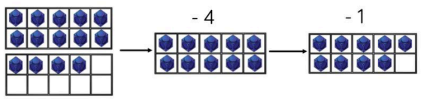
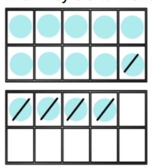


	~~70~~ 60	2 → 12
-	40	7
	20	5

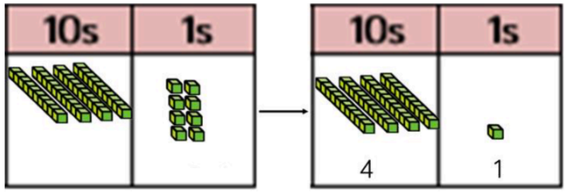
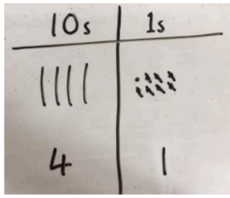
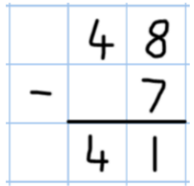
A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

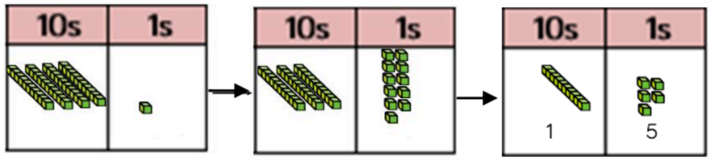
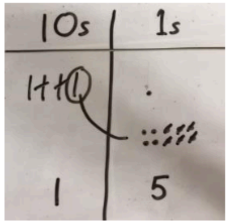
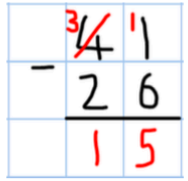
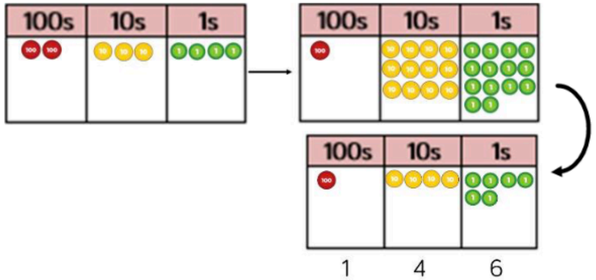
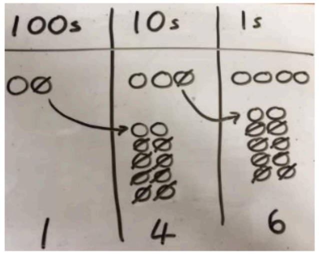
Year 1 Concrete – Pictorial - Abstract

Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = $\boxed{}$ = 4 – 3 
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is $\boxed{}$ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.
Making 10 using ten frames. 14 – 5 	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend. 14 - 5 = 9 4 1 14 – 4 = 10 10 – 1 = 9

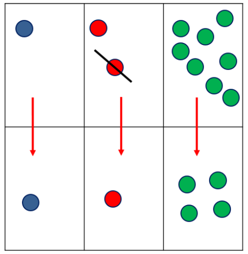
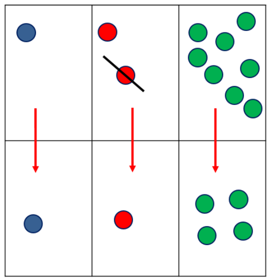
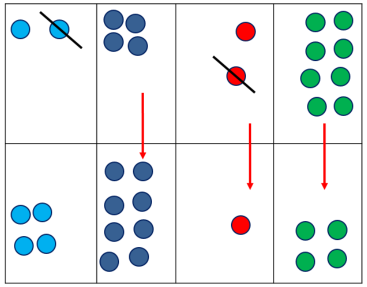
Year 2 Concrete – Pictorial - Abstract

Column method using base 10. 48-7  	Children to represent the base 10 pictorially.  	Column method or children could count back 7. 

Year 3 and beyond Concrete – Pictorial - Abstract

Column method using base 10 and having to exchange. 41 – 26  	Represent the base 10 pictorially, remembering to show the exchange.  	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 
Column method using place value counters. 234 – 88  	Represent the place value counters pictorially; remembering to show what has been exchanged.  	Formal colum method. Children must understand what has happened when they have crossed out digits. 2 1 2~~3~~4 - 88 6

Subtraction
Whole Numbers

Year 4	Year 5	Year 6
Missing number/digit problems: 456 + □ = 710; 1□7 + 6□ = 200; 60 + 99 + □ = 340; 200 – 90 – 80 = □; 225 - □ = 150; □ – 25 = 67; 3450 – 1000 = □; □ - 2000 = 900 **<u>Mental methods</u>** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. **<u>Written methods (progressing to 4-digits)</u>** Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.  $\begin{array}{rrrr} & 200 & \overset{20}{\cancel{30}} & {}^{1}2 \\ - & 100 & 10 & 4 \\ \hline & 100 & 10 & 8 \end{array}$ If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with place value counters.  $\begin{array}{rr} & 2\overset{2}{\cancel{3}}{}^{1}2 \\ - & 114 \\ \hline & 118 \end{array}$	Missing number/digit problems: 6.45 = 6 + 0.4 + □; 119 - □ = 86; 1 000 000 - □ = 999 000; 600 000 + □ + 1000 = 671 000; 12 462 – 2 300 = □ **<u>Mental methods</u>** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. **<u>Written methods (progressing to more than 4-digits)</u>** When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.  $\begin{array}{rr} & \overset{5}{\cancel{6}}{}^{1}2\overset{2}{\cancel{3}}{}^{1}2 \\ - & 4814 \\ \hline & 1418 \end{array}$ Progress to calculating with decimals, including those with different numbers of decimal places.	Missing number/digit problems: □ and # each stand for a different number. # = 34. # + # = □ + □ + #. What is the value of □? What if # = 28? What if # = 21 10 000 000 = 9 000 100 + □ 7 – 2 x 3 = □; (7 – 2) x 3 = □; (□ - 2) x 3 = 15 **<u>Mental methods</u>** should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. **<u>Written methods</u>** As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured. Teachers may also choose to introduce children to other efficient written layouts which help develop conceptual understanding. For example: $\begin{array}{r} 326 \\ \underline{-148} \\ -2 \\ -20 \\ \underline{200} \\ \underline{178} \end{array}$ Continue calculating with decimals, including those with different numbers of decimal places.

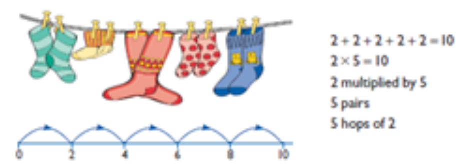
Multiplication

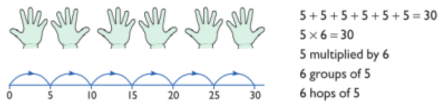
Whole Numbers

Year 1

Understand multiplication is related to doubling and combing groups of the same size (repeated addition)

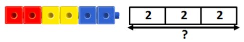
Washing line, and other practical resources for counting. Concrete objects. Numicon; bundles of straws, bead strings



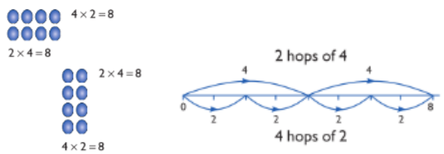


Problem solving with concrete objects (including money and measures

Use cubes and and bar method to develop the vocabulary relating to 'times' –
Pick up two, 3 times



Use arrays to understand multiplication can be done in any order (commutative)



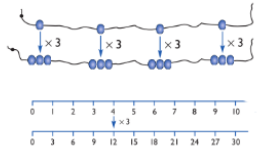
Year 2

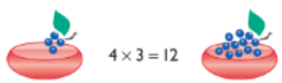
Expressing multiplication as a number sentence using x
Using understanding of the inverse and practical resources to solve missing number problems.

7 x 2 =   = 2 x 7
7 x   = 14   14 =   x 7
  x 2 = 14   14 = 2 x
  x ◯ = 14   14 =   x ◯

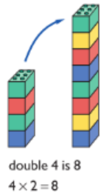
Develop understanding of multiplication using array and number lines (see Year 1). Include multiplications not in the 2, 5 or 10 times tables.

Begin to develop understanding of multiplication as scaling (3 times bigger/taller)



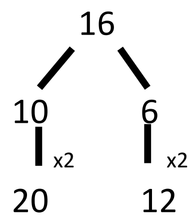


Doubling numbers up to 10 + 10
Link with understanding scaling
Using known doubles to work out double 2d numbers
(double 15 = double 10 + double 5)



Towards written methods

Use jottings to develop an understanding of doubling two digit numbers.



Year 3

Missing number problems
Continue with a range of equations as in Year 2 but with appropriate numbers.

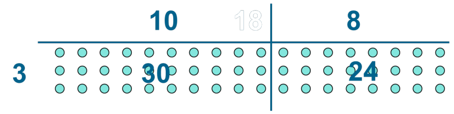
Mental methods
Doubling 2 digit numbers using partitioning

Demonstrating multiplication on a number line – jumping in larger groups of amounts

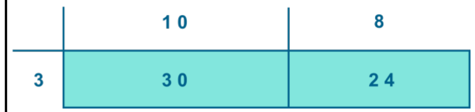
13 x 4 = 10 groups 4 = 3 groups of 4

Written methods (progressing to 2d x 1d)

Developing written methods using understanding of visual images

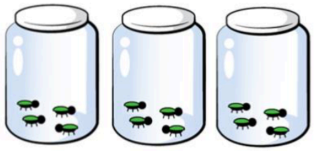
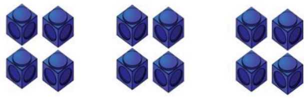
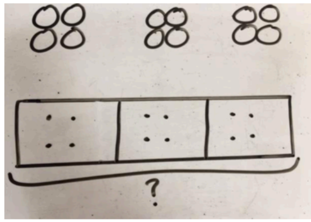


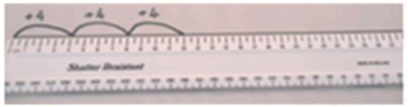
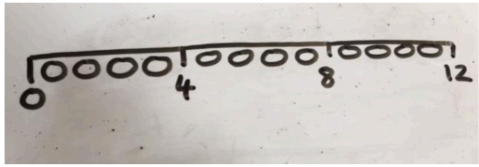
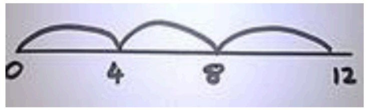
Develop onto the grid method

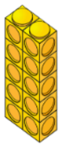
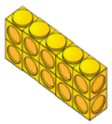
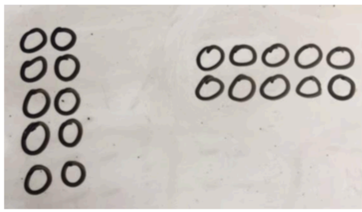


Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters

Year 1 and 2 Concrete – Pictorial - Abstract

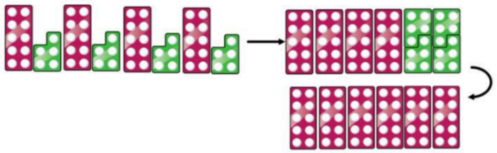
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group.  	Children to represent the practical resources in a picture and use a bar model.  	3 × 4 = 12 4 + 4 + 4 = 12

Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.:  	Abstract number line showing three jumps of four. 3 × 4 = 12 

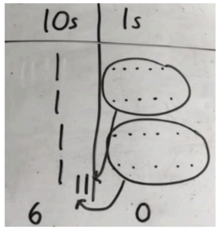
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5  5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

Year 3 Concrete – Pictorial - Abstract

Partition to multiply using Numicon, base 10 or Cuisenaire rods.
4 × 15



Children to represent the concrete manipulatives pictorially.

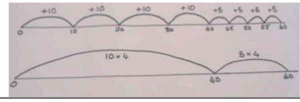


Children to be encouraged to show the steps they have taken.

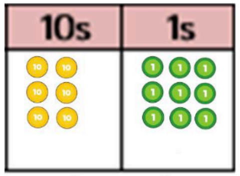
4 × 15
10 5

10 × 4 = 40
5 × 4 = 20
40 + 20 = 60

A number line can also be used

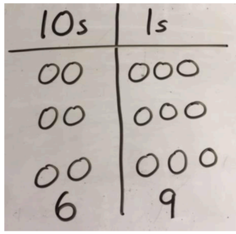


Formal column method with place value counters (base 10 can also be used.) 3 × 23



6 9

Children to represent the counters pictorially.

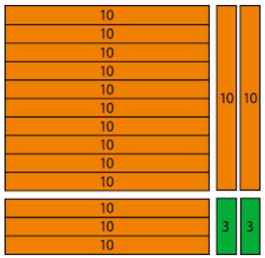


Children to record what it is they are doing to show understanding.

3 × 23
20 3

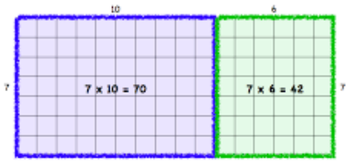
3 × 20 = 60
3 × 3 = 9
60 + 9 = 69

$$\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \\ \hline \end{array}$$



7 x 16 =

7 x (10 + 6) =



70 + 42 = 112

×	30	5
7	210	35

210 + 35 = 245

Multiplication
Whole Numbers

Year 4

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

2 x 5 = 160

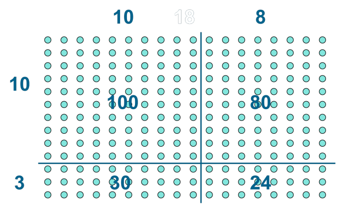
Mental methods

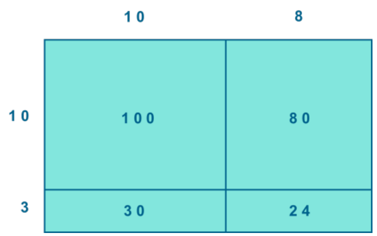
Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

Written methods (progressing to 3d x 2d)

Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.





Year 5

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

Mental methods

X by 10, 100, 1000 using moving digits ITP

Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35)

Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning)

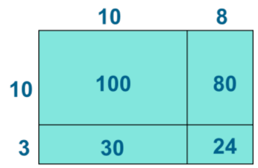
Solving practical problems where children need to scale up. Relate to known number facts.

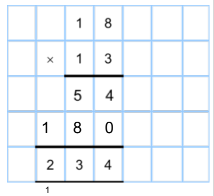
Identify factor pairs for numbers

Written methods (progressing to 4d x 2d)

Long multiplication using place value counters

Children to explore how the grid method supports an understanding of long multiplication (for 2d x 2d)





Year 6

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

Mental methods

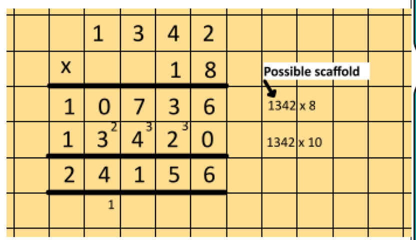
Identifying common factors and multiples of given numbers

Solving practical problems where children need to scale up. Relate to known number facts.

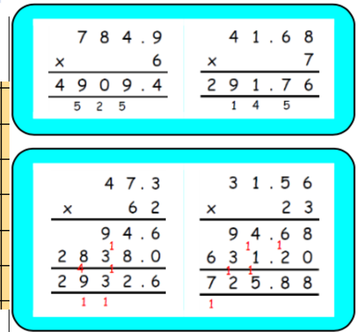
Written methods

Continue to refine and deepen understanding of written methods including fluency for using long multiplication

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16



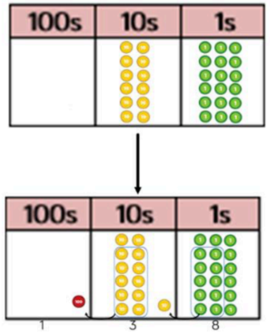
Also include:



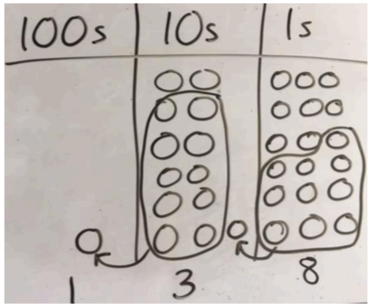
Decimal multiplication in the context of money and measures

Year 4 Concrete – Pictorial - Abstract

Formal column method with place value counters.
6 x 23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

6 x 23 =

$$\begin{array}{r} 23 \\ \times \quad 6 \\ \hline 138 \\ \hline {\scriptstyle 1\ 1} \end{array}$$

Years 5 & 6 Concrete – Pictorial - Abstract

When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

$$\begin{array}{rrrr} & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline & 7_{1} & 4_{2} & 4 \\ 2 & 4 & 8 & 0 \\ \hline 3 & 2 & 2 & 4 \\ \hline {\scriptstyle 1} & {\scriptstyle 1} & & \end{array}$$

Answer: 3224

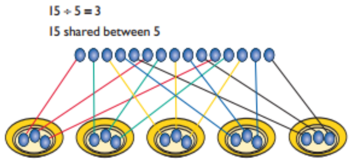
Division
Whole Numbers

Year 1

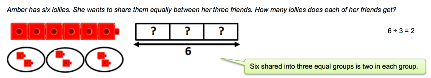
Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.
Children should be given opportunities to reason about what they notice in number patterns.

Group AND **share small quantities- understanding the difference between the two concepts.**

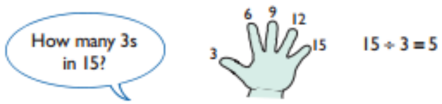
Sharing
Develops importance of one-to-one correspondence.



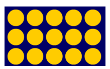
Children should be taught to share using concrete apparatus.

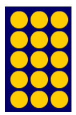


Grouping
Children should apply their counting skills to develop some understanding of grouping.

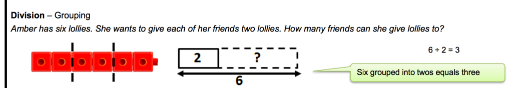


Use of arrays as a pictorial representation for division.
15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.





Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.



Year 2

÷ = signs and missing numbers
6 ÷ 2 =   = 6 ÷ 2
6 ÷   = 3   3 = 6 ÷
  ÷ 2 = 3   3 =   ÷ 2
  ÷ ∇ = 3   3 =   ÷ ∇

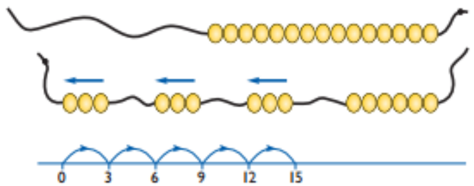
Know and understand sharing and grouping- introducing children to the ÷ sign.

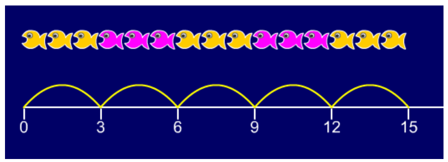
Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Grouping using a numberline

Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'.

15 ÷ 3 = 5



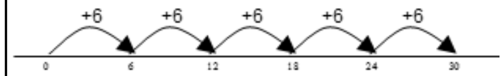


Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

Year 3

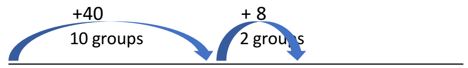
÷ = signs and missing numbers
Continue using a range of equations as in year 2 but with appropriate numbers.

Grouping
How many 6's are in 30?
30 ÷ 6 can be modelled as:

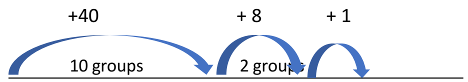


Becoming more efficient using a numberline

Children need to be able to partition the dividend in different ways.
48 ÷ 4 = 12



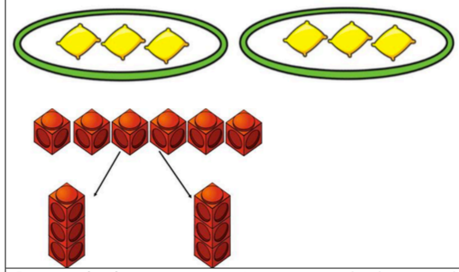
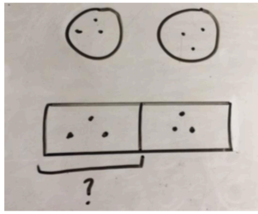
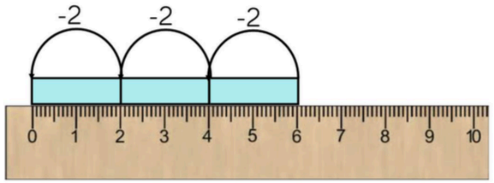
Remainders
49 ÷ 4 = 12 r1

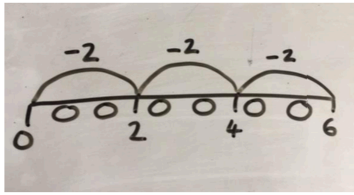
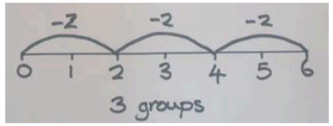


Sharing – 49 shared between 4. How many left over?
Grouping – How many 4s make 49. How many are left over?

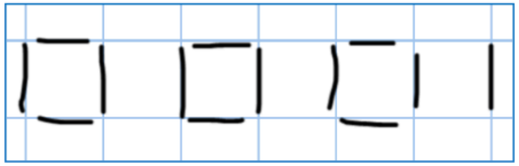
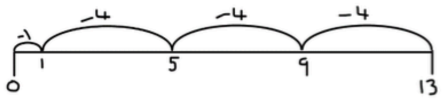
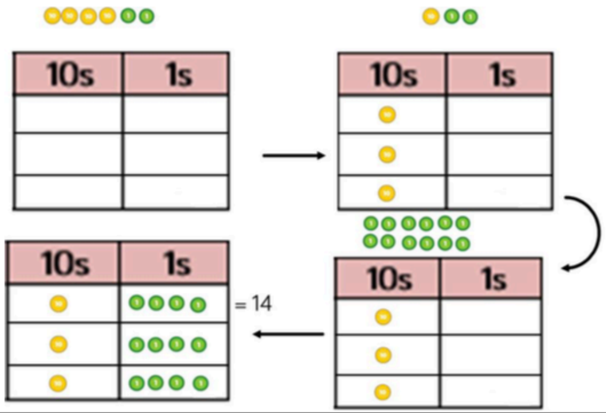
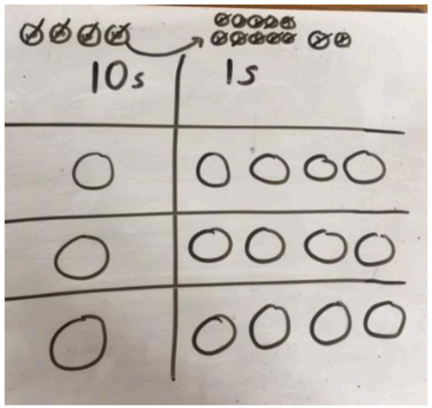
Place value counters can be used to support children apply their knowledge of grouping.
For example:
60 ÷ 10 = How many groups of 10 in 60?
600 ÷ 100 = How many groups of 100 in 600?

Year 1 and 2 Concrete – Pictorial - Abstract

<table>
<tr>
<td>Sharing using a range of objects.
6 ÷ 2
</td>
<td>Represent the sharing pictorially.

</td>
<td>6 ÷ 2 = 3
<table><tr><td>3</td><td>3</td></tr></table>
Children should also be encouraged to use their 2 times tables facts.</td>
</tr>
<tr>
<td>Repeated subtraction using Cuisenaire rods above a ruler.
6 ÷ 2


3 groups of 2</td>
<td>Children to represent repeated subtraction pictorially.

</td>
<td>Abstract number line to represent the equal groups that have been subtracted.

</td>
</tr>
</table>

Year 3: Concrete – Pictorial - Abstract

2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 – 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Division
Whole Numbers

Year 4 / Year 5

<u>÷ = signs and missing numbers</u>

Continue using a range of equations as in year 3 but with appropriate numbers.

<u>Sharing, Grouping and using a number line</u>

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:

1. Dividend just over 10x the divisor, e.g. 84 ÷ 7
2. Dividend just over 10x the divisor when the divisor is a teen number, e.g. 173 ÷ 15 (learning sensible strategies for calculations such as 102 ÷ 17)
3. Dividend over 100x the divisor, e.g. 840 ÷ 7
4. Dividend over 20x the divisor, e.g. 168 ÷ 7

All of the above stages should include calculations with remainders as well as without.

Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)

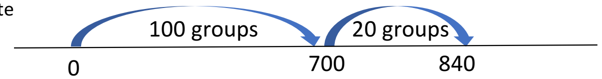
e.g. 840 ÷ 7 = 120

<u>Jottings</u>

7 x 100 = 700

7 x 10 = 70

7 x 20 = 140

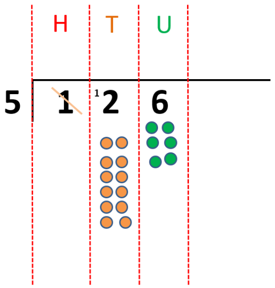


Year 4

<u>Formal Written Methods</u>

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1

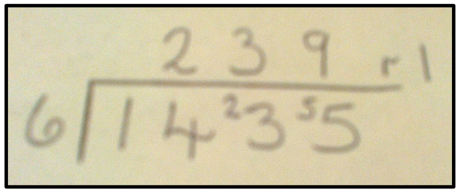


Year 5

<u>Formal Written Methods</u>

Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used (see link from fig. 1 in Year 4)

E.g. 1435 ÷ 6



Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Year 6

<u>÷ = signs and missing numbers</u>

Continue using a range of equations but with appropriate numbers

<u>Sharing and Grouping and using a number line</u>

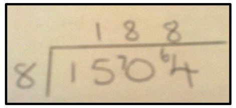
Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.

Quotients should be expressed as decimals and fractions

Children will divide to two decimal places.

<u>Formal Written Methods – short division</u>

E.g. 1504 ÷ 8



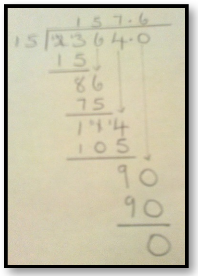
```
      2 3 . 3 3 3
24 ) 5 6 0 . 0 0 0
      (carries: 5 8 8 8 8)
```

<u>Formal Written Methods – long division (chunking)</u> *link to number line*

```
13 | 1 9 3 7
   - 1 3 0 0     13 x 100
       6 3 7
   -   5 2 0     13 x 40
       1 1 7
   -   1 1 7     13 x 9
           0
```

<u>Formal Written Methods – long division</u>

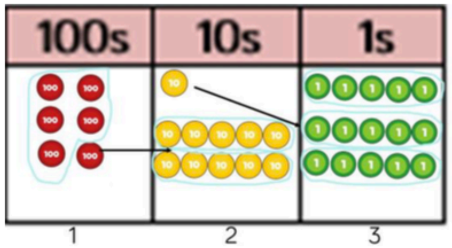
E.g. 2364 ÷ 15



```
        2 8 . 8
15 | 4 3 2 . 0
     3 0
     1 3 2
     1 2 0
       1 2 0
       1 2 0
           0
```

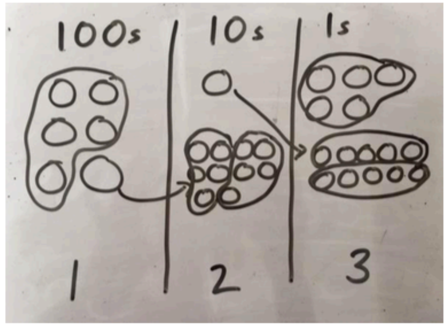
Year 4: Concrete – Pictorial - Abstract

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

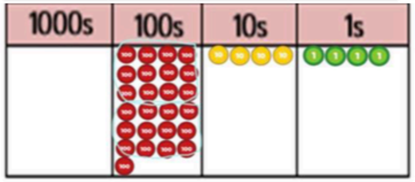
$$\begin{array}{r} 123 \\ 5\,\overline{\smash{)}\,6\,{}^{1}1\,{}^{1}5} \end{array}$$

Year 5 & 6: Concrete – Pictorial - Abstract

Long division using place value counters

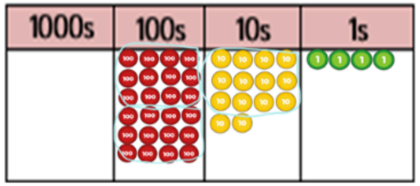
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



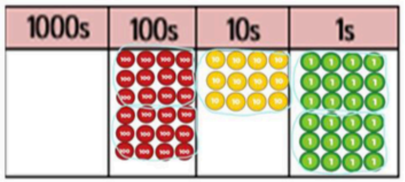
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\,\overline{)\,2544} \\ 24 \\ \hline 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\,\overline{)\,2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



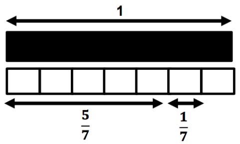
After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

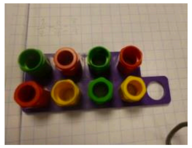
$$\begin{array}{r} 0212 \\ 12\,\overline{)\,2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Addition and Subtraction with Fractions

Year 3

Make with Cuisenaire rods

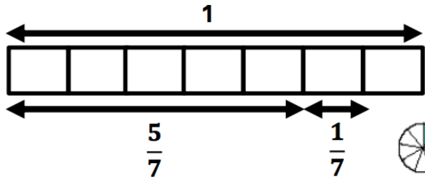




$2/9 + 6/9 = 8/9$

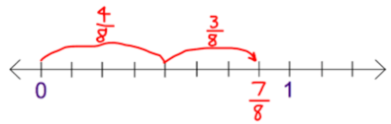
$6/6 - 2/6 = 4/6$

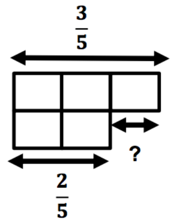
Bar Model



$\frac{2}{9} + \frac{2}{9}$

Number line



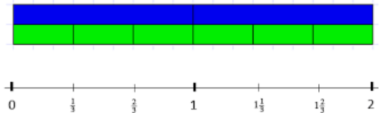


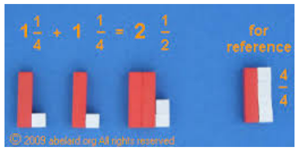
$$\frac{3}{5} + \frac{1}{5} =$$

$$\frac{5}{7} - \frac{1}{7} =$$

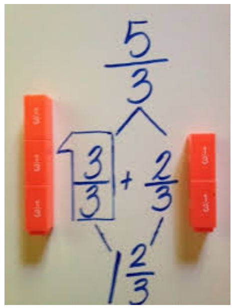
Year 4

Counting in fractional steps using concrete objects and on a number line beyond 1 is an important prerequisite to calculating with fractions.

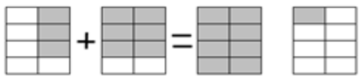




1.3 or 1 3/10 or 13/10.



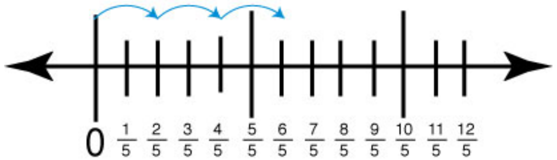
Bar Model



$$\frac{3}{8} + \frac{6}{8} = \frac{9}{8} = 1\frac{1}{8}$$

Number line

$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}$$

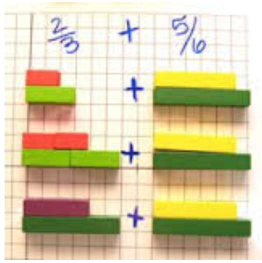
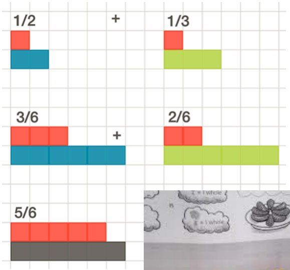
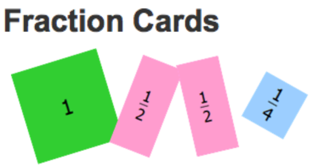
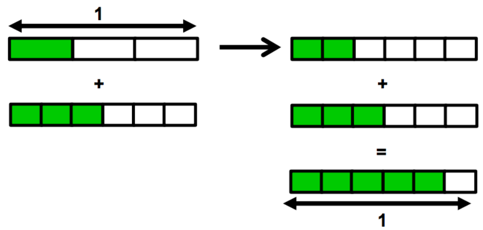
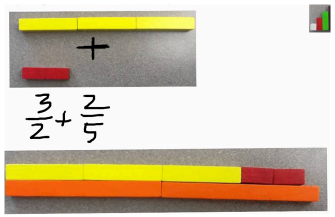
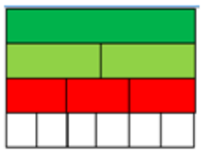
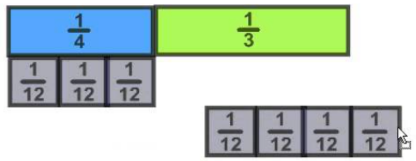


$$\frac{4}{7} + \frac{5}{7} =$$

$$\frac{16}{15} - \frac{12}{15} =$$

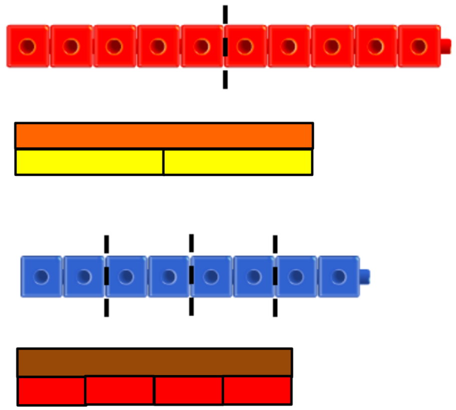
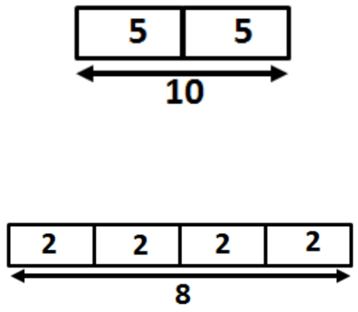
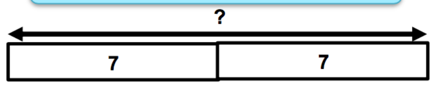
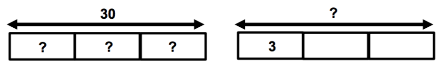
$$\frac{62}{100} - \frac{38}{100} =$$

Addition and Subtraction with Fractions

Year 5 Children must be confident with finding equivalent fractions before they start adding and subtracting fractions with different denominators.	   		$2\frac{1}{3}+\frac{5}{6}=$ $\frac{3}{4}-\frac{3}{8}=$ $\frac{3}{10}-\frac{1}{20}=$
Year 6	  $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$ $=\frac{5}{6}$		$\frac{1}{4}+\frac{1}{5}+\frac{1}{10}=$ $1\frac{1}{4}-\frac{1}{3}=$

Concrete resources should be used to support transition or practice at any point if children are unsure.

Multiplying with Fractions

Year 1		 	
Year 2	*Half*  What fractions do you see? If the orange is worth 6, what is the value of the yellow? If each of the yellow rods is worth 8, what is the value of the orange? If the orange is worth 100, what is the value of the yellow? *Thirds*  	Half of a number is 7, what is the number?   	

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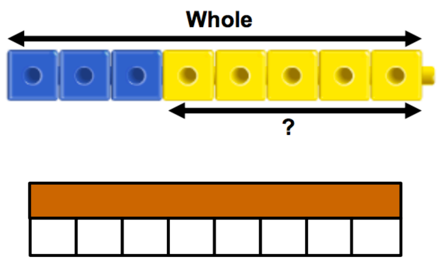
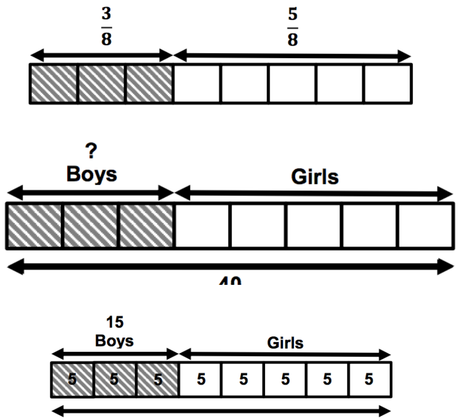
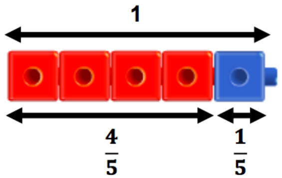
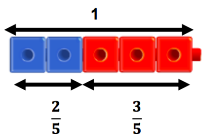
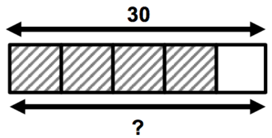
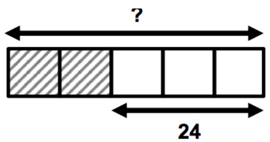
Multiplying with Fractions

Year 3 Finding fractions of objects	$\frac{3}{8}$ *of a class are boys. What fraction of the class are girls?* 		
Year 4	*Sally buys four fifths of the shop's apples. If the shop had 30 apples, how many apples did she buy?*  *James had some football cards. He gave two fifths away. He now has 24 cards. How many did he have to start with?* 		Method: Divide the whole number by the denominator, and multiply by the numerator. This will have been supported by the bar model.

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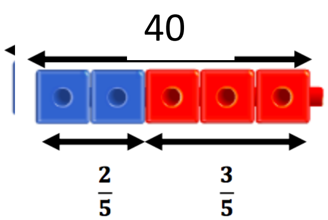
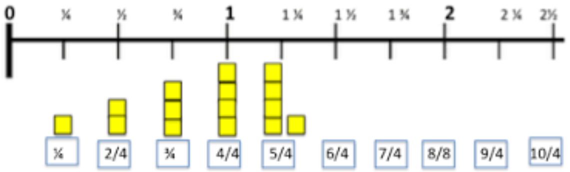
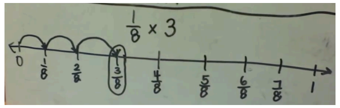
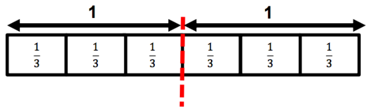
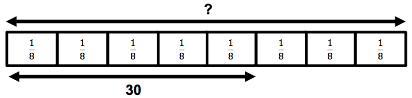
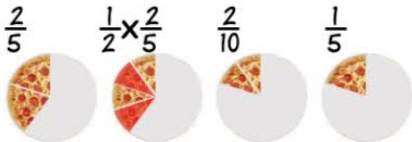
Multiplying with Fractions

Year 5 Children should be fully confident with simplifying fractions and converting improper fractions to mixed numbers.	 	 $6 \times \frac{1}{3} = ?$  $= \frac{6}{3} = 2$ 	Method: Multiply the whole number by the numerator and divide by the denominator. **Children must understand this as multiplying the number of parts and then expressing as a mixed number.** **This will have been supported by the number line.**
Year 6 This approach builds on the concept of finding a fraction **of**.	 $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ 	 **Array**	**Multiply the numerators** **Multiply the denominators** **Simplify is necessary** $\frac{3}{5} \times \frac{6}{7} = \frac{3 \times 6}{5 \times 7} = \frac{18}{35}$

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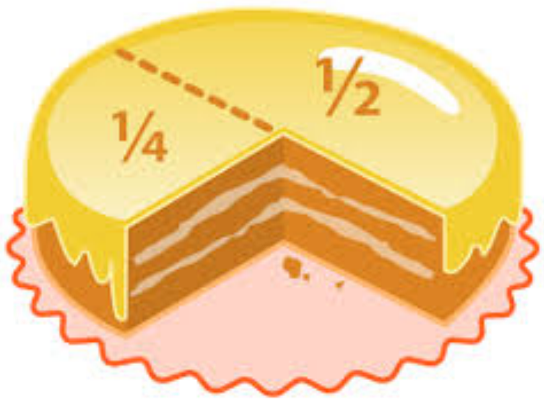
Dividing Fractions

Year 6

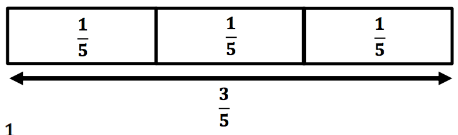
Children should make the link that finding a fraction of is the same as dividing by whole number.

It is important for children to see dividing "fractions" first and foremost as dividing a set of things. E.g. 3 out of 8 divided by 3 = 1 out of 8.
A strip of paper can be used to represent a fraction and can be folded into a number of pieces to represent the division. Then the number of parts can be worked out. E.g. if half a cake is split between four people, this would be 8 parts out of the whole cake.

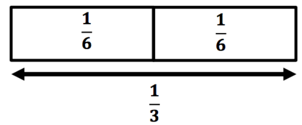


Bar Model

$\frac{3}{5} \div 3 =$



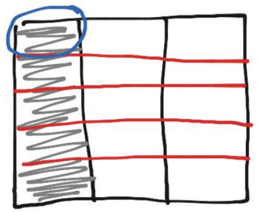
$\frac{1}{3} \div 2 =$



Array

After a birthday party, 1/3 of a large cake was left. If this leftover cake was shared equally among the 5 people, what fraction of the whole cake did each person receive?

$\frac{1}{3} \div 5 = \frac{1}{15}$



Method:
Multiply the denominator by the whole number.
Simplify if necessary.

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